VOLTAGE-FREQUENCY UPDATE FOR NONLINEAR ANALYSIS OF FREE-RUNNING AND INJECTION-LOCKED MULTIPLE DEVICE OSCILLATORS

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ABSTRACT

The standard voltage update algorithm is modified for use in oscillator problems in which the frequency may be unknown and the embedding circuit is a high-impedance resonant circuit. The method is then applied to large-signal steady-state analysis of strongly coupled power combining oscillator circuits under free-running and externally injection-locked conditions. Tunability, power output, locking, and sensitivity to device parameters are examined.

INTRODUCTION

Highly developed general purpose nonlinear analysis programs, based on optimization or continuation based strategies for solving harmonic balance equations, are now widely available. However, in many cases, for example when studying a specific class of circuits or when using specially programmed device models, it is still advantageous to write application-specific nonlinear analysis programs. In this paper we describe enhancements to the voltage-update (relaxation) technique [1] which enable it to be efficiently applied to multiple-device power-combining oscillators. Due to the large signal levels and strong interaction between devices in these structures, self-consistent nonlinear analysis is a necessity in understanding their tuning, locking, and device sensitivity characteristics.

Relaxation techniques have several disadvantages compared to optimization or continuation methods [2,3]. However, most of the disadvantages can be overcome with a few additions to the basic algorithm. In this paper we describe (1) the use of time-domain updating to reduce computation time, (2) the addition of frequency updating to allow analysis of free-running oscillators, and (3) simple ways to change the splitting of the harmonic balance equations to enable reliable convergence in high-impedance resonant embedding circuits.

There are several features which make voltage update attractive. It generally requires more iterations than a continuation or optimization algorithm, but each iteration is much faster and utilizes less memory, since no gradient calculations are required. The advantage is particularly apparent when the number of harmonics and devices is large. Additionally, complex device models which require numerical integration or series expansions often produce some amount of numerical roughness. Perturbation-type gradient calculations are sensitive to this type of small-scale error, while relaxation is not. Finally, when developing new programs, relaxation is much simpler to code and debug.

METHOD

Basic Voltage Update Method For periodic (single-tone) problems, an initial estimate for the voltage across each port joining a linear and nonlinear subcircuit is expressed both as a Fourier series with coefficients $V^1_0,\ldots,V^1_M$, and through the use of a transform matrix, as a time series $v^1_1,\ldots,v^1_N$, where $N = 2M+1$. The voltage is then applied to the nonlinear subcircuit to find the current into it; this current is subsequently applied to the linear subcircuit to find a set of voltages $V^{1'}_0,\ldots,V^{1'}_M$ and $v^{1'}_1,\ldots,v^{1'}_N$. A new estimate for the voltages is found from

$$V^{i+1}_j = (1 - P_j)V^i_j + P_j V^{i'}_j \qquad j = 0,\ldots,M.$$

in the frequency domain, where the P_j are relaxation constants, and the process is repeated until convergence occurs.

Time Domain Updating The update can instead be carried out in the time domain:

$$v^{i+1}_j = (1 - P_j)v^i_j + P_j v^{i'}_j \qquad j = 1,\ldots,N.$$

In the usual method, the time-domain samples are applied during the nonlinear part of the iteration, the resulting current is transformed to the frequency domain and applied to the linear circuit, and then transformed back to the time domain. If the frequency is not variable, however, the iterations can be performed entirely in the time domain without explicitly using the Fourier coefficients. This is done by forming a single matrix A which directly converts the vector of time samples for the current (i^i) into the time samples for the voltage $v^{i'}$:

$$v^{i'} = Ai^i$$

where $A = FZF^{-1}$, F is the Fourier transform matrix, and Z is a banded diagonal matrix containing the linear subcircuit impedance at each harmonic. In multiple-device circuits F becomes a block diagonal matrix, each block Fourier-transforming the voltage at one device port, and Z becomes a multiple banded matrix, with the off-diagonal bands due to coupling between the ports. If the frequency is known and fixed A needs to be formed only once for a given problem.

Frequency Update In problems such as free-running oscillators where there is no driving source, the frequency is also a variable. If the iteration process has arrived at a correct solution, the frequency components of $V^{i'}$ are identical to the components of V^i, and thus the phases of corresponding components are also identical. If there is a small error $\Delta\omega$ in the frequency, the phases of the corresponding components of V^i and $V^{i'}$ will be different, with the magnitude roughly increasing with the error in frequency. This suggests using this phase to find the frequency. At each iteration, we

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calculate the phase change between $V^{i'}$ and V^{i} for the largest frequency component and update the frequency according to

$$\omega^{i+1} = \omega^{i}\left[1 - p_w \Delta\phi/\pi\right]$$

where $\Delta\phi$ is the phase change and p_w is a relaxation constant. The optimum magnitude and sign of p_w is dependent on the specific circuit, but convergence can be obtained for a wide range of values; a typical value would be 1/2Q where Q is the quality factor of the linear embedding circuit.

When frequency update is used, the time-domain updating scheme described in the previous section is not efficient since the matrix A would have to be recalculated at every iteration due to the changing frequency.

Convergence Properties A well-known difficulty in the update method is that the convergence properties of a problem are closely related to the impedance levels [1]; a detailed discussion is given in [2,pp.101-104]. For voltage-update it is best for the linear port impedances to be less than those at the nonlinear ports; if this is not the case convergence may not be possible for any positive value of the relaxation constants, regardless of how close the guess is to the true solution. One solution if the impedance levels are not suitable is to update currents rather than voltages [1]. However, this requires evaluation of voltage as a function of current for the nonlinear circuit, which can be difficult for realistic voltage-controlled devices.

A simpler way to improve the reliability is to add a pair (R,-R) of non-physical resistors (or, more generally, impedances Z,-Z) across each port joining a nonlinear subcircuit to a linear one, as shown in Figure 1. The pair draws no net current and therefore will not perturb the final solution for the voltage. One resistor is considered to be part of each of the subcircuits. By correctly choosing the resistors, the impedances levels and polarities of the two halves can be adjusted to ensure convergence with reasonably large values for the relaxation constants. Mathematically, this is equivalent to changing the splitting of the nonlinear equations to make them more suitable for relaxation, as suggested in [2].

In our algorithm the added resistors are chosen such that the resulting nonlinear impedance is larger than (about twice) the linear and the real parts have the same sign. Of course, the impedance of the nonlinear subcircuit is not known precisely in advance. In principle the resistances could be assigned adaptively during the iteration process as more accurate impedances are found, and could be different at each harmonic; however, in practice we have found that fixed values, based on the initial guess information, are sufficient. The addition of these resistors is *not* analagous to the artificial reduction of Q often used to find an approximate solution in time domain simulations of oscillators; the voltage in our case remains exactly the same as in the unmodified circuit.

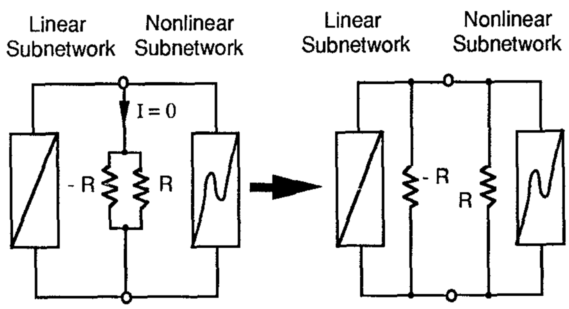

***Figure 1**: Addition of non-physical resistors to circuit to improve convergence. There is no effect on the final solution for the voltages in the circuit.*

Convergence Results For single device cube-law oscillators in RLC circuits, with 12 harmonics considered, and with the impedances adjusted as described above, values of $0.1<P<0.5$ and p_w about 0.5/Q typically give amplitude convergence to 1% in 20 iterations and 0.01% in 40 iterations. The addition of more harmonics does not significantly increase the number of iterations required. The frequency typically converges to at least 6 digits in 40 iterations.

However, there is still an element of uncertainty due to the need to choose the relaxation constants and compensating resistors. Some trial and error may be necessary to find parameters in the correct range for any given class of circuits, and for this reason we recommend our algorithm only for specific application programs and not as a replacement for optimization or gradient based methods in general purpose nonlinear software.

Application to Free-Running Oscillator To analyze a free-running oscillator, an initial guess is made for the amplitude and frequency of oscillation. The voltages and frequency are updated at each iteration as described above. During the iteration process the phasor values of the fundamental coefficient resemble Figure 2(A). If the circuit does not support oscillation then the solution will tend toward zero as shown in Figure 2(C). As in other harmonic-balance methods, there is a danger of false convergence to zero a.c. amplitude since there is usually an unstable zero-solution satisfying the harmonic balance equations.

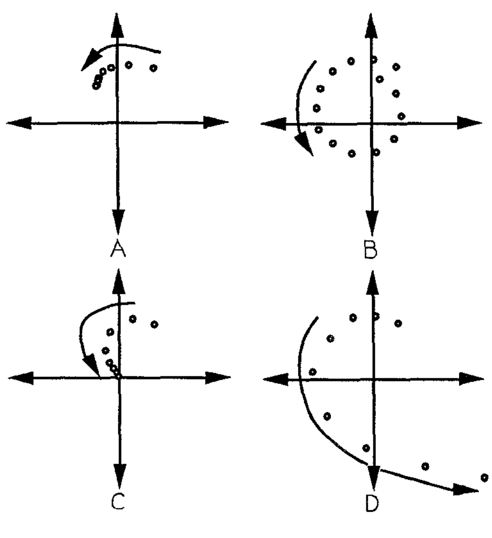

***Figure 2**: Change of complex amplitude of fundamental during iteration. **A**: Forced-frequency case, or free-running case where frequency is updated using the method described above. The iterations converge to a stable non-zero solution. **B**: A free-running case in which the frequency is incorrect and is not updated ($p_w = 0$). The iterations converge to a steady non-zero amplitude with rotating phase. **C**: A case where there is no oscillation. The iterations converge toward zero. **D**: Divergent case*

Application to Injection-Locked Oscillator In this case, one applies time-domain updating without frequency-update, with the addition of an injection source as part of the linear circuit. The solution is assumed to contain only harmonics (or possibly subharmonics) of the injection signal. If an injection-locked solution exists, the algorithm converges to it as in Figure 2(A). If the circuit instead oscillates in a free-running condition, the behavior illustrated in Figure 2(B) is exhibited: the amplitude goes to a slowly varying steady-state, with the phase changing by a near-constant amount at each iteration. The same behavior is seen in a no-injection oscillator when the assumed frequency is incorrect and not updated.

This procedure only indicates the presence or absence of a locked solution; it can not distinguish between hold-in range and pull-in range. To find other possible states, multitone (almost-periodic) analysis would have to be performed. The ill-conditioning difficulties associated with such analyses are well known ([3], [4]) and many methods for treating them have been proposed. The problems here would be especially severe because one frequency is variable (requiring new time sample points and transform matrices at each iteration), and the variable frequency may become close or even identical to the injection frequency, thus magnifying the ill-conditioning.

PERIODIC POWER-COMBINING OSCILLATORS

Power-combining circuits consisting of active devices placed periodically along a transmission line or cavity have been studied extensively [4]; recently, planar versions of this structure utilizing Gunn diodes periodically shunting a microstrip line have been tested [5]. Typically the devices are placed one-half wavelength apart as shown in Figure 3, with the load placed at one end and an open or shorted section at the opposite end. The device capacitances are individually resonated by means of parallel inductive stubs at each device node, with the resonant frequency near that which satisfies the half-wave spacing condition. The proper value for the load Z decreases with the number of devices.

Because of the strong coupling, the ability of the devices to self-lock to each other cannot be accurately treated using the small-injection formulation of Adler. The eigenvector treatment given in [4] is very useful for examining the modal properties of the structure, but does not include the effects of higher harmonics or the frequency dependence of the large signal impedances. Harmonic balance solves the circuit equations self-consistently, and can be easily applied when the oscillation frequency is tuned away from the center (half-wave periodic) frequency of the structure and the different devices no longer operate under identical conditions.

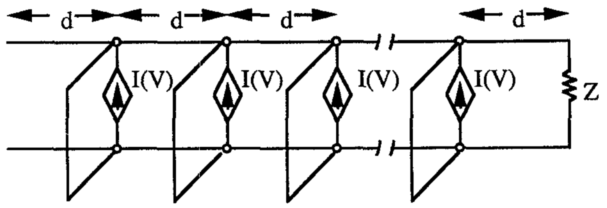

Figure 3: Schematic of power-combining oscillator. In a typical circuit, d *is approximately one-half wavelength. The devices in this case are Gunn diodes, each with a resonating inductive stub.*

Application of Method to Multiple-Device Oscillator In the case of multiple device oscillators, each of the voltage harmonics at each of the device ports is considered a state variable, so that the number of variables is now multiplied by the number of devices. At each iteration the currents are found through all the devices and these currents are then applied to the impedance matrix for the entire linear circuit to find a new set of voltages at the ports. Multiple-device circuits require more conservative values for P and thus converge slower than single-device circuits.

Since we have not included multitone signals in our formulation, we can only find "locked" (all devices synchronized) solutions, if no solution is found, as in Figure 2(B), this indicates that the devices may be oscillating independently of each other. Voltage-frequency update is used to find free-running (self-locked) solutions; voltage update alone is used to find externally injection-locked states.

Example Calculation Results For the examples here 10-mW class X-band Gunn diodes are modeled by a simple cubic law $i = a_1 v + a_2 v^2 + a_3 v^3$ in parallel with a capacitance of 0.4 pF, with $a_1 = -0.01$, $a_2 = 0.0015$, and $a_3 = 0.001$. The algorithm has been used without difficulty with devices (SIS diodes) exhibiting much more complicated behavior, including time-history dependent nonlinear reactance. The diodes are placed along an ideal 50-ohm transmission line, with a load of 111/N ohms at the end, where N is the number of diodes. A shorted stub is placed in parallel with each device to resonate the capacitance. Twelve harmonics were carried through all the computations.

Figures 4 through 6 show the effects of tuning the oscillator away from its center frequency by changing the resonant frequency of the capacitance-stub combination at each diode. The oscillation frequency is determined partially by the periodicity of the structure and partially by the individual device resonances, as shown in Figure 4. As the number of diodes increases, the importance of the periodicity decreases relative to the effect of the individual resonances. For large deviations from the center frequency, no solution is found, indicating that the diodes do not lock to each other.

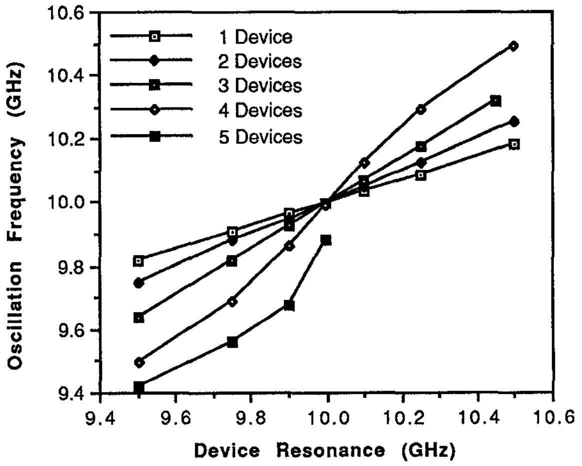

Figure 4: Variation of oscillation frequency with changes in the resonant frequencies of the individual devices; for one, two, three, four, and five diode power combiners. Device spacing is one-half wavelength at 10 GHz.

Figure 5 shows the change in output power as the oscillator is tuned. In this example, the power decreased slightly in the one-diode case but increased in the multiple-diode cases. One possible explanation is that the devices see different impedances as the oscillator is detuned. One diode produces more power, and tends to become a "master" oscillator, injection-locking the others. Depending on the specific circuit, the injection-locking may increase or decrease the power produced from the other devices. This point of view is supported by examining the power distribution in a three-diode power combiner as the frequency is varied (Figure 6). When the structure is not half-wave periodic the diodes have different operating points and generate different power levels.

The external injection-locking properties of the power-combiner were studied by injecting a fixed-frequency current from a source located at the end of the current opposite to the load. Figure 7 shows the locking bandwidth as a function of injection level for a single diode and three-diode circuits. The locking bandwidth is reduced in the multiple-device oscillator.

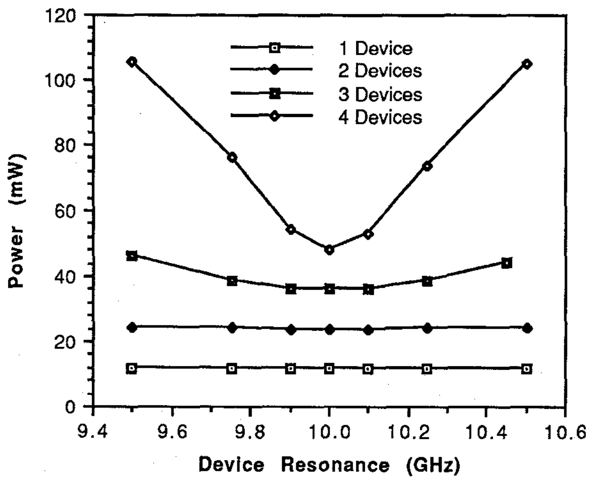

Figure 5: *Variation of output power as resonant frequencies of individual devices are varied. Devices are one-half wavelength apart at 10 GHz.*

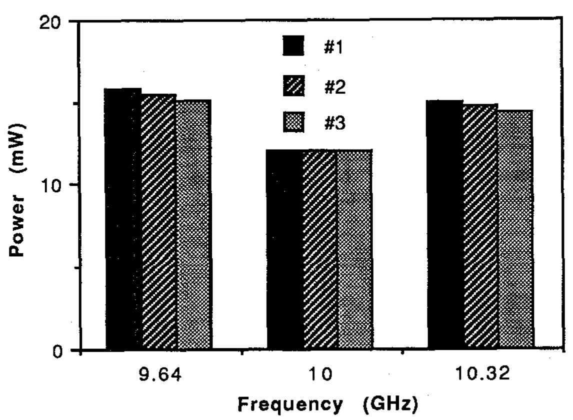

Figure 6: *Change in power produced by each device in a three-diode power combiner, as frequency is varied. Devices are one-half wavelength apart at 10 GHz.*

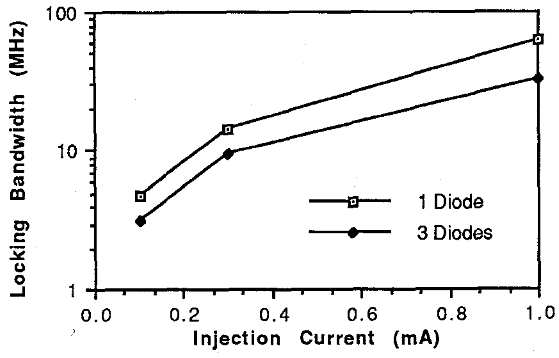

Figure 7: *The deflection from center frequency that can be achieved by an external injection-locking source, as a function of injection current.*

CONCLUSIONS

Although the need to select relaxation constants and convergence improvement resistances makes it less competitive for general-purpose programs, the relaxation technique is attractive for application-specific nonlinear analysis programs. We have described simple improvements to make the technique more efficient in fixed frequency problems, more applicable to variable frequency problems, and more reliable in its convergence properties.

With the modifications described, voltage-update has proven to be effective in the analysis of multiple-device, strongly-coupled circuits. Information can be obtained about the relative effects of device tuning versus periodicity of the circuit, power variation with tuning, power distribution among the devices, self-locking characteristics, and injection-locking characteristics.

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